Purpose

*Focus is on drag optimization to maximize rocket performance!*
Agenda

- Definitions
- Mission Parameters
- Nose Cone Design
- Fin Design
- Summary
- Appendices
- References & Web Sites
Definitions

- **Drag Coefficient**
  - **Parasitic Drag**
    - **Form/Pressure/Profile Drag**
      - Dependence upon the profile of the object
      - **Base Drag**
        - Due to Boundary Layer separation at base of airframe/fins
    - **Skin Friction (Viscous) Drag**
      - Friction of the fluid against the skin of the object
    - **Interference Drag**
      - Incremental drag above sum of all other drag components. Created at protrusion intersections.
  - **Induced (Lift-Induced) Drag**
    - Due to redirection of airflow
  - **Wave (Compressibility) Drag**
    - Due to shockwaves when moving near or above the speed of sound (typically leading & trailing edges)
  - **Rotational Drag**
    - Circumferential velocity from roll will thicken boundary layer and result in increased drag
Definitions

- **Wetted Area**
  - Surface Area exposed to airflow

- **Fineness (Aspect) Ratio**
  - Nose Cone Length/Base Diameter

- **Bluffness Ratio**
  - Tip Diameter/Base Diameter
  - Hemispherical Blunting
  - Me’plat Diameter is a Flat Truncation (e.g., bullets and artillery shells)
Definitions¹

- **Laminar Boundary Layer**
  - Fluid streams move in parallel (negligible transfer of momentum)

- **Turbulent Boundary Layer**
  - Fluid streams transverse with velocity variations around an average value

- **Boundary Layer Separation**
  - Boundary layer separates from object’s surface creating an effective profile

- **Reynolds Number**
  - Dimensionless ratio of inertial / viscous forces
  - [http://www.grc.nasa.gov/WWW/BGH/reynolds.html](http://www.grc.nasa.gov/WWW/BGH/reynolds.html)
Definitions

• Aspect Ratio (AR)
  • Fin Span / Average Fin Cord

• Effective Aspect Ratio
  • Working AR due to Airflow Effects

• Taper Ratio
  • Tip Cord / Root Cord
Definitions

• **Thrust Profile**
  • Thrust vs. Time Curve

• **Velocity Definitions**
  • Subsonic: < .8 Mach
  • Transonic: .8 to 1.2 Mach
  • Supersonic: 1.2 to 5 Mach
  • Hypersonic: > 5 Mach
Mission Parameters

- **Velocity**
  - Coefficient of Drag
  - Thrust Profile
  - Total Mass
- **Altitude**
  - Coefficient of Drag
  - Thrust Profile
  - Total and Coasting Mass
- **Mass**
  - Material Volume and Strength
  - Payload
- **Payload**
  - Available Volume
  - Stability Impacts
- **Stability (CP&CG - Discussed Last Year)**
Nose Cone Design

- **Mission Dependent Variables**
  - Payload
  - Stability (CP, CG)
- **Independent Variables**
  - Atmospheric Density
  - Temperature
  - Wind Conditions
  - Surface Finish
  - Angle of Attack
Nose Cone Design

• Assumptions
  • Zero Angle of Attack
  • Constant Surface Finish
  • No Roll
  • No Aerodynamic Heating Effects
### Nose Cone Solutions

#### Best in Class

<table>
<thead>
<tr>
<th>Category</th>
<th>1.</th>
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<td><strong>X(^{1/2}) Power Series</strong></td>
<td><strong>X(^{1/2}) Power Series</strong></td>
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<td><strong>Supersonic</strong></td>
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<td><strong>Love Minimum Drag</strong></td>
<td><strong>X(^{6}) Power Series</strong></td>
<td><strong>X(^{6}) Power Series</strong></td>
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</tbody>
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Fineness Ratio$^{6,7}$

- Increasing Fineness Ratio
  - Decreases Wave Drag
  - Increases Skin Friction Drag
  - Optimum Ratio is approximately 5
Bluntness Ratio\(2,3,5\)

- **Optimal ratio is** .15
  - Provided length remains constant
- **Applicability dependent upon fineness ratio and velocity**
  - Fineness ratio \(\leq 5\)
  - Below Hypersonic
Coefficient of Drag ($C_D$) Subsonic$^1$

- Primarily Skin Friction Drag
- Minimal Pressure Drag
- No Wave Drag
- No Interference Drag
- No Induced Drag
- Elliptical
  - Fineness Ratio of 2
Coefficients of Drag ($C_D$) — Transonic

Wave Drag Increases Substantially
Pressure Drag becomes Significant
Fineness Ratio of 5 is Critical

Comparison of drag characteristics of various nose shapes in the transonic-to-low Mach regions. Rankings are: superior (1), good (2), fair (3), inferior (4).

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Coefficient of Drag ($C_D$) Supersonic

- Pressure Drag Decreases
- Wave Drag Decreases
- Fineness Ratio of 5 is Critical
Coefficient of Drag \( (C_D) \) Hypersonic\(^{8,9,10}\)

- \( x^{0.6} \) Power Series
  - Fineness Ratio of 5 or 6
- Varies with Fineness Ratio
- No Blunting

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Fin Design

- Mission Dependence
  - Stability (CP, CG, Roll, ...)
- Independent Variables
  - Atmospheric Density
  - Temperature
  - Wind Conditions
  - Surface Finish (Assumed Constant)
  - Angle of Attack (Assumed Zero)
Fin Optimization

- Minimize Drag
- Maintain Structural Integrity
  - Minimize Divergence
  - Minimize Bending-Torsion Flutter
  - Minimize Mass
- Maximize Fin Joint Strength
- Maintain Passive Stability
Fin Drag Optimization

• No General Solution Unearthed
  • Computational Models Exist at Subsonic, Transonic, and Supersonic Speeds

• Solution Factors
  • Velocity
  • Density
  • Lift Requirements (Corrective Moment) at Angles of Attack
  • ...

• Structural Strength
Fin Count

- Fin Count > 3
  - Skin Friction
    Drag Increases
  - Interference
    Drag Increases up to Mach 1.35

Fin Count → 3 but not always ...
Fin Tip Vortices

- Vortices alter Fin Effective Aspect Ratio
- Positive or Neutral Ratio Desired
  - Lower Angles of Attack for Given Lift (Increases Corrective and Damping Moments)
  - Lower Induced Drag for Given Lift
- Desire Zero or Positive Effective Aspect Ratio
- Ease of Manufacture
  - Implies Fins with a Tip Cord > 0
  - Square Edge Tips
Fin Flutter²⁰

• NASA Safety Factors
  • 15% between vehicle & flutter velocity
  • 32% between vehicle and flutter dynamic pressure
Fin Flutter

- Stall Flutter not applicable
- Choose Shear Modulus for Material
- Apply Contingency when selecting Flutter criterion
- Criterion then used with Aspect Ratio to find Thickness Ratio
- Multiple Thickness Ratio & Cord to get Thickness
Fin Joint Drag$^{1,12}$

- **Interference Drag**
  - Minimized when fillet radius is between 4% and 8% of fin root cord
  - 10” Root Cord $\rightarrow$ $\frac{1}{2}$” Radius
  - Consider Structural Strength

- **Wing (Leading Edge) Fillets**
  Increase Drag in the Transonic Region
Sweep Angle

- **70° Sweep Angle Superior to Smaller Angles in Sub, Trans, & Supersonic Ranges**
- **4 Fin Configuration Exception in Subsonic Region**
Fin Thickness\textsuperscript{15,17,18}

- Thinner Symmetrical Fins Result in Lower $C_D$ in Sub, Trans, and Supersonic Regions
Leading Edge$^{14}$

(b) Delta wings.

(b) Trapezoidal wings.

(b) Drag coefficients and lift-drag ratios.
Leading Edge \(^{14}\)

- **At Mach 4**
  - Sharp Leading Edge has Lower \(C_D\) at all Angles of Attack
  - Trapezoidal (Clipped Delta) has Lower \(C_D\) than Delta
Trailing Edge

- Trailing-edge Thickness up to 0.7% Root Cord Reduces Transonic Drag
  - Does not Impact Subsonic Drag
- Trailing Edge Thickness > 0.7% Results in Increased Drag
- Varies with Airfoil Thickness and Optimum is < 0.7%
  - 10” Root Cord → $\frac{1}{16}$” Thick Trailing Edge
Fin Cross Section\textsuperscript{13,19}

- Sub, Trans, and Supersonic
  - Hexagonal Lower $C_D$ than Double Wedge
- Supersonic
  - $C_D$ NACA 65A003 < 65A004 < Hexagonal
Shape$^{14,19}$

- **Supersonic Data**
  - Trapezoidal (Clipped Delta) Lower $C_D$ than Delta
  - Delta and Diamond have Similar $C_D$
Multi-Disciplinary Design Optimization (MDO)\textsuperscript{x}

- Optimizing Individual Components may not Result in an Optimum Design
  - Increasing Fin count from 3 to 4
  - Improving Nose Cone Fineness Ratio (3.5 vs. 7) may Result in Increased Fin Drag at Some Velocities
Summary

- **Optimal Nose Cones**
  - Subsonic – Elliptical
  - Transonic – Von Karman (Blunted 15% of Base Diameter)
  - Supersonic - $X^{3/4}$ Power Series
  - Hypersonic – $X^6$ Power Series
  - Fineness Ratio of 5

- **Fin Optimization**
  - Fin Count of 3
  - Fin Joints 4% to 8% of Root Cord
  - Thickness < 10% of Root Cord often between 3% & 6%
  - Trailing Edge Flat but < 0.7% of Root Cord in Thickness
  - Leading Edge may be Sharp
  - Sweep Angle between 45° and 70°
  - Flat Fin Tips
  - Hexagonal Cross Section
  - Clipped Delta Shape
Appendices

Nose Cones
Nose Cone Geometries

• Conical
• Elliptical
• Ogive (Tangent)
• Parabolic
• Power Series
• Sears-Haack (Von Karman)
Nose Cone Parameters

- \( L \) is the overall length of the nosecone
- \( R \) is the radius of the base of the nosecone
- \( y \) is the radius at any point \( x \), as \( x \) varies from 0 at the tip of the nosecone to \( L \)
- The full body of revolution of the nosecone is formed by rotating the profile around the centerline \( (C_L) \)

Dimensions used in the equations

- \( x = 0 \)
- \( y = 0 \)
- \( x = L \)
- \( y = R \)
Conical Nose Cones

- The sides of a cone are straight lines, so the diameter equation is simply, \( y = \frac{Rx}{L} \)
- Cones are sometimes defined by their ‘half angle’, \( \phi = \tan^{-1}\left(\frac{R}{L}\right) \) and \( y = x \tan \phi \)
- \( C_p = \frac{L}{3} \)
- \( V = \pi R^2 L / 3 \)
- \( S = \pi R (R^2 + L^2)^{\cdot5} \)
Elliptical Nose Cones

• The profile of this shape is one-half of an ellipse, with the major axis being the centerline and the minor axis being the base of the nosecone.
• This shape is advantageous for subsonic flight due to its blunt nose and tangent base.
• It is defined by: \( y = R(1-x^2/L^2)^{1/2} \)
• \( C_p = \frac{3L}{2} \)
• \( V = 2\pi R^2 L/3 \)
• \( S = \pi L^2 + \left[ \pi R^2 / \sigma \ln \left( \frac{1+\sigma}{1-\sigma} \right) \right] / 2 \) where \( \sigma = \frac{L^2 + R^2}{L} \)
Tangent Ogive Nose Cones

- This shape is formed by a circle segment where the base is on the circle radius and the airframe is tangent to the curve of the nosecone at its base.
- The radius of the circle that forms the ogive is: $\rho = \frac{(R^2 + L^2)}{2R}$
- The radius $y$ at any point $x$, as $x$ varies from 0 to $L$ is: $y = (\rho^2 - (x - L)^2)^{1/2} + R - \rho$ where $L \leq \rho$
- $C_p = \frac{V}{\pi R^2}$
- $V = \pi [L\sigma^2 - L^3/\sigma - (\sigma^3 - R\sigma^3)\sin^{-1}(L/\sigma)]$ where $\sigma = \frac{(R^2 + L^2)}{2R}$
- $S = ?$
The Parabolic Series nose shape is generated by rotating a segment of a parabola around a line parallel to its axis of symmetry.

$$y = R\{(2[\pi/L] - K[\pi/L]^2)/(2-K)\} \text{ for } 0 \leq K \leq 1$$

- \(K = 0\) for a CONE
- \(K = 0.5\) for a 1/2 PARABOLA
- \(K = 0.75\) for a 3/4 PARABOLA
- \(K = 1\) for a PARABOLA (base tangent to airframe)

- \(C_p = \frac{L}{2}\)
- \(V = \pi R^2 L/2\)
- \(S = R^2/4L\)
Power Series Nose Cones

• The Power Series shape is characterized by its (usually) blunt tip, and by the fact that its base is not tangent to the body tube.
• The Power series nose shape is generated by rotating a parabola about its major axis. The base of the nosecone is parallel to the latus rectum of the parabola, and the factor $n$ controls the ‘bluntness’ of the shape. As $n$ decreases towards zero, the Power Series nose shape becomes increasingly blunt; at values of $n$ above about .7, the tip becomes sharp.
• \( y = R\left(\frac{x}{L}\right)^n \) for \( 0 \leq n \leq 1 \)
  - \( n = 1 \) for a CONE
  - \( n = .75 \) for a \( \frac{3}{4} \) POWER
  - \( n = .5 \) for a \( \frac{1}{2} \) POWER (PARABOLA)
  - \( n = 0 \) for a CYLINDER
• \( C_p = ? \)
• \( V = ? \)
• \( S = ? \)
Sears-Haack Nose Cones

- Not constructed from geometric figures
- Mathematically derived for drag minimization
- Not tangent to body at base
- Rounded not sharp nose tips
- \[ y = R\{\theta-[\sin(2\theta)/2]+C\sin^3(\theta)\}^{1/2}/(\pi)^{1/2} \] where 0≤C and \( \theta = \cos^{-1}(1-2x/L) \)
  - C = 0 minimum drag for given Length and Volume (LV)
  - C = 1/3 minimum drag for given Length and Diameter (LD - Von Karman)
- \( C_p = L^{1/2} \) Von Karman; \( C_p = .437L \) LV-Haack
- V=?
- S=?
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